DPP - Daily Practice Problems

Chapter-wise Sheets

Date :   **Start Time :**   **End Time :**

PHYSICS   CP08

SYLLABUS : Mechanical Properties of Solids

Max. Marks : 180   **Marking Scheme :** (+4) for correct & (–1) for incorrect answer   **Time : 60 min.**

INSTRUCTIONS : This Daily Practice Problem Sheet contains 45 MCQs. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.
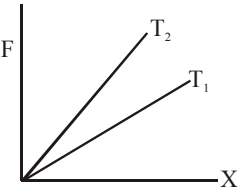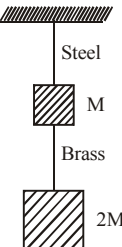
1. Two wires A and B are of the same material. Their lengths are in the ratio 1 : 2 and the diameter are in the ratio 2 : 1. If they are pulled by the same force, then increase in length will be in the ratio
(a) 2 : 1   (b) 1 : 4   (c) 1 : 8   (d) 8 : 1

2. The load versus elongation graphs for four wires of same length and made of the same material are shown in the figure. The thinnest wire is represented by the line

(a) OA

(b) OC

(c) OD

(d) OB

3. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is
(a) 16 J   (b) 8 J   (c) 32 J   (d) 24 J

4. A metal wire of length L_1 and area of cross-section A is attached to a rigid support. Another metal wire of length L_2 and of the same cross-sectional area is attached to the free end of the first wire. A body of mass M is then suspended from the free end of the second wire. If Y_1 and Y_2 are the Young's moduli of the wires respectively, the effective force constant of the system of two wires is

(a) $\dfrac{(Y_1 Y_2) A}{2(Y_1 L_2 + Y_2 L_1)}$ (b) $\dfrac{(Y_1 Y_2) A}{(L_1 L_2)^{1/2}}$

(c) $\dfrac{(Y_1 Y_2) A}{Y_1 L_2 + Y_2 L_1}$ (d) $\dfrac{(Y_1 Y_2)^{1/2} A}{(L_2 L_1)^{1/2}}$

5. The approximate depth of an ocean is 2700 m. The compressibility of water is 45.4×10^{-11} Pa^{-1} and density of water is 10^3 kg/m^3. What fractional compression of water will be obtained at the bottom of the ocean ?
(a) 1.0×10^{-2} (b) 1.2×10^{-2}
(c) 1.4×10^{-2} (d) 0.8×10^{-2}

6. The Young's modulus of steel is twice that of brass. Two wires of same length and of same area of cross section, one of steel and another of brass are suspended from the same roof. If we want the lower ends of the wires to be at the same level, then the weights added to the steel and brass wires must be in the ratio of :
(a) 2 : 1   (b) 4 : 1   (c) 1 : 1   (d) 1 : 2

7. Choose the wrong statement.
(a) The bulk modulus for solids is much larger than for liquids.
(b) Gases are least compressible.
(c) The incompressibility of the solids is due to the tight coupling between neighbouring atoms.
(d) The reciprocal of the bulk modulus is called compressibility.

| **RESPONSE GRID** | **1.** ⓐⓑⓒⓓ | **2.** ⓐⓑⓒⓓ | **3.** ⓐⓑⓒⓓ | **4.** ⓐⓑⓒⓓ | **5.** ⓐⓑⓒⓓ |
| | **6.** ⓐⓑⓒⓓ | **7.** ⓐⓑⓒⓓ | | | |

Space for Rough Work

8. A copper wire of length 1.0 m and a steel wire of length 0.5 m having equal cross-sectional areas are joined end to end. The composite wire is stretched by a certain load which stretches the copper wire by 1 mm. If the Young's modulii of copper and steel are respectively 1.0×10^{11} Nm^{-2} and 2.0×10^{11} Nm^{-2}, the total extension of the composite wire is :

(a) 1.75 mm (b) 2.0 mm (c) 1.50 mm (d) 1.25 mm

9. A cube at temperature 0ºC is compressed equally from all sides by an external pressure P. By what amount should its temperature be raised to bring it back to the size it had before the external pressure was applied. The bulk modulus of the material of the cube is B and the coefficient of linear expansion is a.

(a) P/B α (b) P/3 B α (c) 3 π α/B (d) 3 B/P

10. The diagram below shows the change in the length X of a thin uniform wire caused by the application of stress F at two different temperatures T_1 and T_2. The variation shown suggests that

(a) $T_1 > T_2$

(b) $T_1 < T_2$

(c) $T_2 > T_1$

(d) $T_1 \geq T_2$

11. If the ratio of lengths, radii and Young's moduli of steel and brass wires in the figure are *a*, *b* and *c* respectively, then the corresponding ratio of increase in their lengths is :

(a) $\dfrac{3c}{2ab^2}$

(b) $\dfrac{2a^2c}{b}$

(c) $\dfrac{3a}{2b^2c}$

(d) $\dfrac{2ac}{b^2}$

12. The Young's modulus of brass and steel are respectively 10^{10} N/m^2. and 2×10^{10} N/m^2. A brass wire and a steel wire of the same length are extended by 1 mm under the same force, the radii of brass and steel wires are R_B and R_S respectively. Then

(a) $R_S = \sqrt{2} \, R_B$ (b) $R_S = R_B / \sqrt{2}$

(c) $R_S = 4R_B$ (d) $R_S = R_B / 4$

13. Steel ruptures when a shear of 3.5×10^8 N m^{-2} is applied. The force needed to punch a 1 cm diameter hole in a steel sheet 0.3 cm thick is nearly:

(a) 1.4×10^4 N   (b) 2.7×10^4 N
(c) 3.3×10^4 N   (d) 1.1×10^4 N

14. A ball falling in a lake of depth 400 m has a decrease of 0.2% in its volume at the bottom. The bulk modulus of the material of the ball is (in N m^{-2})

(a) 9.8×10^9 (b) 9.8×10^{10}
(c) 1.96×10^{10} (d) 1.96×10^9

15. A circular tube of mean radius 8 cm and thickness 0.04 cm is melted up and recast into a solid rod of the same length. The ratio of the torsional rigidities of the circular tube and the solid rod is

(a) $\dfrac{(8.02)^4 - (7.98)^4}{(0.8)^4}$ (b) $\dfrac{(8.02)^2 - (7.98)^2}{(0.8)^2}$

(c) $\dfrac{(0.8)^2}{(8.02)^4 - (7.98)^4}$ (d) $\dfrac{(0.8)^2}{(8.02)^3 - (7.98)^2}$

16. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area *A* and wire 2 has cross-sectional area 3*A*. If the length of wire 1 increases by Δx on applying force *F*, how much force is needed to stretch wire 2 by the same amount?

(a) $4F$ (b) $6F$ (c) $9F$ (d) F

17. In materials like aluminium and copper, the correct order of magnitude of various elastic modului is:

(a) Young's modulus < shear modulus < bulk modulus.
(b) Bulk modulus < shear modulus < Young's modulus
(c) Shear modulus < Young's modulus < bulk modulus.
(d) Bulk modulus < Young's modulus < shear modulus.

18. What per cent of length of wire increases by applying a stress of 1 kg weight/mm^2 on it?
($Y = 1 \times 10^{11}$ N/m^2 and 1 kg weight = 9.8 newton)
(a) 0.0067%   (b) 0.0098%
(c) 0.0088%   (d) 0.0078%

19. An elastic string of unstretched length *L* and force constant *k* is stretched by a small length *x*. It is further stretched by another small length *y*. The work done in the second stretching is :

(a) $\dfrac{1}{2}ky^2$ (b) $\dfrac{1}{2}k(x^2 + y^2)$

(c) $\dfrac{1}{2}k(x + y)^2$ (d) $\dfrac{1}{2}ky(2x + y)$

20. Two, spring *P* and *Q* of force constants k_p and $k_Q \left(k_Q = \dfrac{k_p}{2} \right)$ are stretched by applying forces of equal magnitude. If the energy stored in *Q* is E, then the energy stored in *P* is

(a) E   (b) 2E   (c) E/2   (d) E/4

| **RESPONSE GRID**	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ
	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ
	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ		

21. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100ºC is:

(For steel Young's modulus is $2 \times 10^{11}\,\text{Nm}^{-2}$ and coefficient of thermal expansion is $1.1 \times 10^{-5}\,\text{K}^{-1}$)

(a) 2.2×10^8 Pa   (b) 2.2×10^9 Pa

(c) 2.2×10^7 Pa   (d) 2.2×10^6 Pa

22. A steel ring of radius r and cross sectional area A is fitted onto a wooden disc of radius R (R > r). If the Young's modulus of steel is Y, then the force with which the steel ring is expanded is

(a) A Y (R/r)   (b) A Y (R−r)/r

(c) (Y/A)[(R−r)/r]   (d) Y r/A R

23. Two wires A and B of same material and of equal length with the radii in the ratio 1 : 2 are subjected to identical loads. If the length of A increases by 8 mm, then the increase in length of B is

(a) 2 mm   (b) 4 mm   (c) 8 mm   (d) 16 mm

24. A material has poisson's ratio 0.50. If a uniform rod of it suffers a longitudinal strain of 2×10^{-3}, then the percentage change in volume is

(a) 0.6   (b) 0.4   (c) 0.2   (d) Zero

25. The upper end of a wire of diameter 12mm and length 1m is clamped and its other end is twisted through an angle of 30°. The angle of shear is

(a) 18°   (b) 0.18°   (c) 36°   (d) 0.36°

26. The pressure on an object of bulk modulus B undergoing hydraulic compression due to a stress exerted by surrounding fluid having volume strain $\left(\dfrac{\Delta V}{V}\right)^2$ is

(a) $B^2\left(\dfrac{\Delta V}{V}\right)$ (b) $B\left(\dfrac{\Delta V}{V}\right)^2$

(c) $\dfrac{1}{B}\left(\dfrac{\Delta V}{V}\right)$ (d) $B\left(\dfrac{\Delta V}{V}\right)$

27. A structural steel rod has a radius of 10 mm and length of 1.0 m. A 100 kN force stretches it along its length. Young's modulus of structural steel is $2 \times 10^{11}\,\text{Nm}^{-2}$. The percentage strain is about

(a) 0.16%   (b) 0.32%   (c) 0.08%   (d) 0.24%

28. A beam of metal supported at the two edges is loaded at the centre. The depression at the centre is proportional to
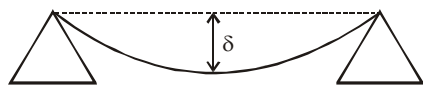
(a) Y^2 (b) Y   (c) 1/Y   (d) $1/Y^2$

29. When a 4 kg mass is hung vertically on a light spring that obeys Hooke's law, the spring stretches by 2 cms. The work required to be done by an external agent in stretching this spring by 5 cms will be (g = 9.8 m/sec²)

(a) 4.900 joule   (b) 2.450 joule

(c) 0.495 joule   (d) 0.245 joule

30. The length of a metal is ℓ_1 when the tension in it is T_1 and is ℓ_2 when the tension is T_2. The original length of the wire is

(a) $\dfrac{\ell_1 + \ell_2}{2}$ (b) $\dfrac{\ell_1 T_2 + \ell_2 T_1}{T_1 + T_2}$

(c) $\dfrac{\ell_1 T_2 - \ell_2 T_1}{T_2 - T_1}$ (d) $\sqrt{T_1 T_2 \ell_1 \ell_2}$

31. For the same cross-sectional area and for a given load, the ratio of depressions for the beam of a square cross-section and circular cross-section is

(a) $3 : \pi$ (b) $\pi : 3$ (c) $1 : \pi$ (d) $\pi : 1$

32. The bulk moduli of ethanol, mercury and water are given as 0.9, 25 and 2.2 respectively in units of $10^9\,\text{Nm}^{-2}$. For a given value of pressure, the fractional compression in volume is $\dfrac{\Delta V}{V}$. Which of the following statements about $\dfrac{\Delta V}{V}$ for these three liquids is correct ?

(a) Ethanol > Water > Mercury

(b) Water > Ethanol > Mercury

(c) Mercury > Ethanol > Water

(d) Ethanol > Mercury > Water
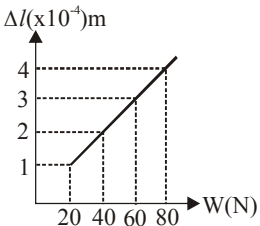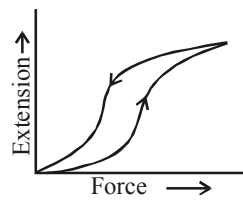
33. The graph given is a stress-strain curve for
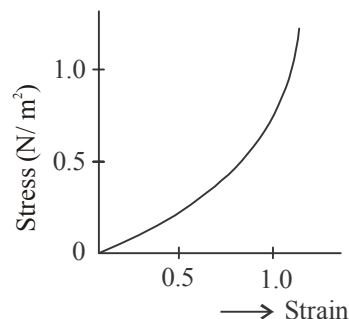
(a) elastic objects   (b) plastics

(c) elastomers   (d) None of these

34. A metal rod of Young's modulus $2 \times 10^{10}\,\text{N m}^{-2}$ undergoes an elastic strain of 0.06%. The energy per unit volume stored in J m^{-3} is

(a) 3600   (b) 7200   (c) 10800   (d) 14400

35. Two wires of the same material and same length but diameters in the ratio 1 : 2 are stretched by the same force. The potential energy per unit volume of the two wires will be in the ratio

(a) 1 : 2   (b) 4 : 1   (c) 2 : 1   (d) 16 : 1

RESPONSE GRID					
21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ	
26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ	
31. ⓐⓑⓒⓓ	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ	

Space for Rough Work

36. The length of an elastic string is a metre when the longitudinal tension is 4 N and b metre when the longitudinal tension is 5 N. The length of the string in metre when the longitudinal tension is 9 N is

(a) $a - b$ (b) $5b - 4a$ (c) $2b - \dfrac{1}{4}a$ (d) $4a - 3b$

37. A force of 10^3 newton, stretches the length of a hanging wire by 1 millimetre. The force required to stretch a wire of same material and length but having four times the diameter by 1 millimetre is

(a) 4×10^3 N (b) 16×10^3 N

(c) $\dfrac{1}{4} \times 10^3$ N (d) $\dfrac{1}{16} \times 10^3$ N

38. A steel wire of length l and cross sectional area A is stretched by 1 cm under a given load. When the same load is applied to another steel wire of double its length and half of its cross section area, the amount of stretching (extension) is

(a) 0.5 cm (b) 2 cm (c) 4 cm (d) 1.5 cm

39. The adjacent graph shows the extension (Δl) of a wire of length 1 m suspended from the top of a roof at one end with a load W connected to the other end. If the cross-sectional area of the wire is 10^{-6} m², calculate the Young's modulus of the material of the wire :

(a) 2×10^{11} N/m² (b) 2×10^{-11} N/m²

(c) 3×10^{-12} N/m² (d) 2×10^{-13} N/m²

40. If a rubber ball is taken at the depth of 200 m in a pool, its volume decreases by 0.1%. If the density of the water is 1×10^3 kg/m³ and g = 10m/s², then the volume elasticity in N/m^2 will be

(a) 10^8 (b) 2×10^8 (c) 10^9 (d) 2×10^9

41. A ball is falling in a lake of depth 200 m creates a decrease 0.1 % in its volume at the bottom. The bulk modulus of the material of the ball will be

(a) 19.6×10^{-8} N/m² (b) 19.6×10^{10} N/m²

(c) 19.6×10^{-10} N/m² (d) 19.6×10^8 N/m²

42. The diagram shows a force-extension graph for a rubber band. Consider the following statements :

I. It will be easier to compress this rubber than expand it

II. Rubber does not return to its original length after it is stretched

III. The rubber band will get heated if it is stretched and released

Which of these can be deduced from the graph:

(a) III only (b) II and III (c) I and III (d) I only

43. The Poisson's ratio of a material is 0.5. If a force is applied to a wire of this material, there is a decrease in the cross-sectional area by 4%. The percentage increase in the length is:

(a) 1% (b) 2% (c) 2.5% (d) 4%

44. Copper of fixed volume 'V; is drawn into wire of length 'l'. When this wire is subjected to a constant force 'F', the extension produced in the wire is 'Δl'. Which of the following graphs is a straight line?

(a) Δl versus $\dfrac{1}{l}$ (b) Δl versus l^2

(c) Δl versus $\dfrac{1}{l^2}$ (d) Δl versus l

45. When a 4 kg mass is hung vertically on a light spring that obeys Hooke's law, the spring stretches by 2 cms. The work required to be done by an external agent in stretching this spring by 5 cm will be (g = 9.8 m/sec²)

(a) 4.900 joule (b) 2.450 joule

(c) 0.495 joule (d) 0.245 joule

Space for Rough Work

1. **(c)** We know that Young's modulus

$$Y = \frac{F}{\pi r^2} \times \frac{L}{\ell}$$

Since Y, F are same for both the wires, we have,

$$\frac{1}{r_1^2} \frac{L_1}{\ell_1} = \frac{1}{r_2^2} \frac{L_2}{\ell_2}$$

or, $$\frac{\ell_1}{\ell_2} = \frac{r_2^2 \times L_1}{r_1^2 \times L_2} = \frac{(D_2/2)^2 \times L_1}{(D_1/2)^2 \times L_2}$$

or, $$\frac{\ell_1}{\ell_2} = \frac{D_2^2 \times L_1}{D_1^2 \times L_2} = \frac{D_2^2}{(2D_2)^2} \times \frac{L_2}{2L_2} = \frac{1}{8}$$

So, $\ell_1 : \ell_2 = 1 : 8$

2. **(a)** From the graph, it is clear that for the same value of load, elongation is maximum for wire *OA*. Hence *OA* is the thinnest wire among the four wires.

3. **(b)** Small amount of work done in extending the spring by *dx* is

$dW = k\, x\, dx$

$$\therefore W = k \int_{0.05}^{0.15} x\, dx$$

$$= \frac{800}{2} \left[(0.15)^2 - (0.05)^2 \right]$$

$= 400\, [(0.15+0.05)(0.15-0.05)]$

$= 400 \times 0.2 \times 0.1 = 8\, J$

4. **(c)** Using the usual expression for the Young's modulus, the force constant for the wire can be written as

$k = \dfrac{F}{\Delta l} = \dfrac{YA}{L}$ where the symbols have their usual meanings. Now the two wires together will have an

effective force constant $\left[\dfrac{k_1 k_2}{k_1 + k_2} \right]$. Substituting the corresponding lengths and the Young's moduli we get the answer.

5. **(b)** Compressibility of water,
$K = 45.4 \times 10^{-11}\, Pa^{-1}$
density of water $P = 10^3\, kg/m^3$
depth of ocean, $h = 2700\, m$

We have to find $\dfrac{\Delta V}{V} = ?$

As we know, compressibility,

$$K = \frac{1}{B} = \frac{(\Delta V/V)}{P}\ (P = \rho gh)$$
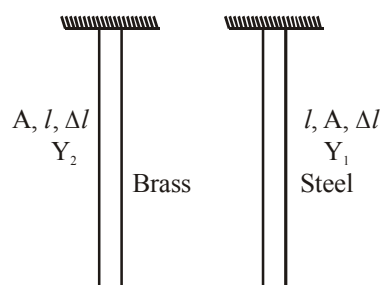
So, $(\Delta V/V) = K\rho gh$
$= 45.4 \times 10^{-11} \times 10^3 \times 10 \times 2700$
$= 1.2258 \times 10^{-2}$

6. **(a)** Young's modulus $Y = \dfrac{W}{A} \cdot \dfrac{l}{\Delta l}$

$$\frac{W_1}{Y_1} = \frac{W_2}{Y_2}$$

$[\because A, l, \Delta l$ same for both brass and steel]

A, l, Δl
Y_2
Brass

l, A, Δl
Y_1
Steel

$$\frac{W_1}{W_2} = \frac{Y_1}{Y_2} = 2 \quad [Y_{steel}/Y_{brass} = 2\ given]$$

7. **(b)** Solids are least compressible whereas gases are highly compressible.

8. **(d)** $Y_c \times (\Delta L_c / L_c) = Y_s \times (\Delta L_s / L_s)$

$$\Rightarrow 1 \times 10^{11} \times \left(\frac{1 \times 10^{-3}}{1} \right) = 2 \times 10^{11} \times \left(\frac{\Delta L_s}{0.5} \right)$$

$$\therefore \Delta L_s = \frac{0.5 \times 10^{-3}}{2} = 0.25\, mm$$

Therefore, total extension of the composite wire =
$\Delta L_c + \Delta L_s$
$= 1\, mm + 0.25\, mm = 1.25\, mm$

9. **(b)** Bulk modulus $B = \dfrac{-P}{(\Delta V/V)} = \dfrac{-PV}{\Delta V}$ ....(1)

and $\Delta V = \gamma V \Delta T = 3\alpha.V.T$ or $\dfrac{-V}{\Delta V} = \dfrac{1}{3\alpha.T.}$ ...(2)

From eqs. (1) and (2), $B = P/(3\alpha.T)$ or $T = \dfrac{P}{3\alpha B}$

10. **(a)** When same stress is applied at two different temperatures, the increase in length is more at higher temperature. Thus $T_1 > T_2$.

11. **(c)** According to questions,

$$\frac{\ell_s}{\ell_b} = a, \frac{r_s}{r_b} = b, \frac{y_s}{y_b} = c, \frac{\Delta \ell_s}{\Delta \ell_b} = ?$$

As, $y = \dfrac{F\ell}{A\Delta\ell} \Rightarrow \Delta\ell = \dfrac{F\ell}{Ay}$

$$\Delta\ell_s = \frac{3mg\ell_s}{\pi r_s^2 \cdot y_s} \quad [\because F_s = (M+2M)g]$$

$$\Delta\ell_b = \frac{2Mg\ell_b}{\pi r_b^2 . y_b} \quad [\because F_b = 2Mg]$$

$$\therefore \frac{\Delta\ell_s}{\Delta\ell_b} = \frac{\dfrac{3Mg\ell_s}{\pi r_s^2 . y_s}}{\dfrac{2Mg.\ell_b}{\pi r_b^2 . y_b}} = \frac{3a}{2b^2 c}$$

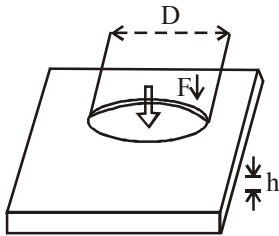**12. (b)** We know that $Y = F L/\pi r^2 \ell$ or $r^2 = F L/(Y \pi \ell)$

$$\therefore R_B^2 = F L/(Y_B \pi \ell) \text{ and } R_S^2 = F L/(Y_S \pi \ell)$$

$$\text{or } \frac{R_B^2}{R_S^2} = \frac{Y_S}{Y_B} = \frac{2\times10^{10}}{10^{10}} = 2$$

$$\text{or } R_B^2 = 2 R_S^2 \text{ or } R_B = \sqrt{2} R_S$$

$$\therefore R_S = R_B / \sqrt{2}$$

13. (c)

Shearing strain is created along the side surface of the punched disk. Note that the forces exerted on the disk are exerted along the circumference of the disk, and the total force exerted on its center only.
Let us assume that the shearing stress along the side surface of the disk is uniform, then

$$F = \int_{surface} dF_{max} = \int_{surface} \sigma_{max} dA = \sigma_{max} \int_{surface} dA$$

$$= \int \sigma_{max} . A = \sigma_{max} . 2\pi\left(\frac{D}{2}\right) h$$

$$= 3.5\times10^8 \times\left(\frac{1}{2}\times10^{-2}\right)\times 0.3\times10^{-2}\times 2\pi$$

$$= 3.297\times10^4 \simeq 3.3\times10^4 \text{ N}$$

14. (d) Bulk modulus is given by, $k = \dfrac{F/A}{\Delta V/V}$

$$= \frac{mg}{A\left(\dfrac{\Delta V}{V}\right)} = \frac{h\rho g}{\left(\dfrac{\Delta V}{V}\right)}, \quad \left(\because \rho = \frac{m}{V}, V = A\times h\right)$$

Given, $h = 400$ m, $\dfrac{\Delta V}{V} = \dfrac{0.2}{100}$

and $\rho = 1\times 10^3$ kg/m^3

$$\therefore k = \frac{400\times10^3 \times 9.8}{0.2/100} = 196\times10^7 \text{ N m}^{-2}$$

$$k = 1.96\times10^9 \text{ N m}^{-2}.$$

15. (a) $C_1 = \dfrac{\pi\eta(r_2^4 - r_1^4)}{2\ell}, C_2 = \dfrac{\pi\eta r^4}{2\ell}$

Initial volume = Final volume

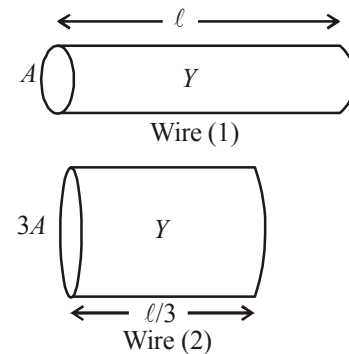$$\therefore \pi[r_2^2 - r_1^2]\ell\rho = \pi r^2 \ell\rho$$

$$\Rightarrow r^2 = r_2^2 - r_1^2 \Rightarrow r^2 = (r_2 + r_1)(r_2 - r_1)$$

$$\Rightarrow r^2 = (8.02 + 7.98)(8.02 - 7.98)$$
$$\Rightarrow r^2 = 16\times 0.04 = 0.64 \text{ cm} \Rightarrow r = 0.8 \text{ cm}$$

$$\therefore \frac{C_1}{C_2} = \frac{r_2^4 - r_1^4}{r^4} = \frac{[8.02]^4 - [7.98]^4}{[0.8]^4}$$

16. (c)

As shown in the figure, the wires will have the same Young's modulus (same material) and the length of the wire of area of cross-section $3A$ will be $\ell/3$ (same volume as wire 1).
For wire 1,
$$Y = \frac{F/A}{\Delta x/\ell} \qquad ...(i)$$
For wire 2,
$$Y = \frac{F'/3A}{\Delta x/(\ell/3)} \qquad ...(ii)$$
From (i) and (ii), $\dfrac{F}{A}\times\dfrac{\ell}{\Delta x} = \dfrac{F'}{3A}\times\dfrac{\ell}{3\Delta x} \Rightarrow F' = 9F$

17. (c) Poisson's ratio, $\sigma = \dfrac{\text{lateral strain }(\beta)}{\text{longitudinal strain }(\alpha)}$

For material like copper, $\sigma = 0.33$
And, $y = 3k(1 - 2\sigma)$

Also, $\dfrac{9}{y} = \dfrac{1}{k} + \dfrac{3}{n}$

$y = 2n(1 + \sigma)$
Hence, $n < y < k$

18. **(b)** Stress $= 1 \text{ kg wt/mm}^2 = 9.8 \text{ N/mm}^2$
$= 9.8 \times 10^6 \text{ N/m}^2$.

$Y = 1 \times 10^{11} \text{N}/\text{m}^2, \qquad \dfrac{\Delta\ell}{\ell} \times 100 = ?$

$Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{\text{Stress}}{\Delta\ell/\ell}$

$\therefore \quad \dfrac{\Delta\ell}{\ell} = \dfrac{\text{Stress}}{Y} = \dfrac{9.8 \times 10^6}{1 \times 10^{11}}$

$\dfrac{\Delta\ell}{\ell} \times 100 = 9.8 \times 10^{-11} \times 100 \times 10^6$

$= 9.8 \times 10^{-3} = 0.0098 \,\%$

19. **(d)** $W_1 = \dfrac{1}{2} kx^2$

and $\qquad W_2 = \dfrac{1}{2} k(x+y)^2$

$\therefore \qquad W = W_2 - W_1 = \dfrac{1}{2} k(x+y)^2 - \dfrac{1}{2}kx^2$

$= \dfrac{1}{2} ky(2x+y)$

20. **(c)** Here, $k_Q = \dfrac{k_p}{2}$

According to Hooke's law
$\therefore \quad F_p = -k_p \, x_p$

$F_Q = -k_Q x_Q \Rightarrow \dfrac{F_p}{F_Q} = \dfrac{k_p}{k_Q} \dfrac{x_p}{x_Q}$

$F_p = F_Q$ [Given]

$\therefore \quad \dfrac{x_p}{x_Q} = \dfrac{k_Q}{k_p}$ ...(i)

Energy stored in a spring is $U = \dfrac{1}{2} kx^2$

$\therefore \quad \dfrac{U_p}{U_Q} = \dfrac{k_p x_p^2}{k_Q x_Q^2} = \dfrac{k_p}{k_Q} \times \dfrac{k_Q^2}{k_p^2} = \dfrac{1}{2} \qquad \left[\because k_Q = \dfrac{k_p}{2} \right]$

$\Rightarrow \quad U_p = \dfrac{U_Q}{2} = \dfrac{E}{2} \qquad\qquad\qquad [\because U_Q = E]$

21. **(a)** Young's modulus $Y = \dfrac{\text{stress}}{\text{strain}}$

stress $= Y \times$ strain
Stress in steel wire = Applied pressure
Pressure = stress = $Y \times$ strain

Strain $= \dfrac{\Delta L}{L} = \alpha \Delta T$ (As length is constant)

$= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100 = 2.2 \times 10^8$ Pa

22. **(b)** Let T be the tension in the ring, then

$Y = \dfrac{T.2\pi r}{A.2\pi (R-r)} = \dfrac{T r}{A (R-r)} \quad \therefore \quad T = \dfrac{Y A (R-r)}{r}$

23. **(a)** Ratio of radii $r_1 : r_2 = 1 : 2$

Ratio of area, $A_1 : A_2 = \pi r_1^2 : \pi r_2^2$

$A_1 : A_2 = 1 : 4$

Now, $\text{Stress}_1 : \text{Stress}_2 = 4 : 1$
So, $\text{Strain}_1 : \text{Strain}_2 = 4 : 1$

$\therefore \quad \dfrac{l_1}{l_2} = \dfrac{4}{1} \Rightarrow 4l_2 = l_1 = 8$

$\therefore \quad l_2 = 2$ mm
Increase in length of B is 2 mm.

24. **(b)** $\dfrac{dV}{V} = (1 + 2\sigma) \dfrac{dL}{L}$

$\dfrac{dV}{V} = 2 \times 2 \times 10^{-3} = 4 \times 10^{-3}$

$\left[\because \sigma = 0.5 = \dfrac{1}{2} \right]$

$\therefore$ Percentage change in volume $= 4 \times 10^{-1} = 0.4\%$

25. **(b)** $r\theta = \ell\phi \Rightarrow \phi = \dfrac{r\theta}{\ell} = \dfrac{6\text{mm} \times 30°}{1\text{m}} = 0.18°$

26. **(d)** Bulk modulus $B = \dfrac{|-dp|}{\left| \left(\dfrac{dV}{V} \right) \right|}$

$\therefore$ Pressure, $dp = B\left(\dfrac{\Delta V}{V} \right)$

27. **(a)** Given: $F = 100 \text{ kN} = 10^5 \text{ N}$
$Y = 2 \times 10^{11} \text{ Nm}^{-2}$
$\ell_0 = 1.0 \text{ m}$
radius $r = 10 \text{ mm} = 10^{-2} \text{ m}$

From formula, $Y = \dfrac{\text{Stress}}{\text{Strain}}$

$\Rightarrow \quad \text{Strain} = \dfrac{\text{Stress}}{Y} = \dfrac{F}{AY}$

$= \dfrac{10^5}{\pi r^2 Y} = \dfrac{10^5}{3.14 \times 10^{-4} \times 2 \times 10^{11}}$

$= \dfrac{1}{628}$

Therefore % strain $= \dfrac{1}{628} \times 100 = 0.16\%$

28. **(c)**

For a beam, the depression at the centre is given by,

$\delta = \left(\dfrac{f L}{4 Y b d^3} \right)$

[f, L, b, d are constants for a particular beam]

i.e. $\delta \propto \dfrac{1}{Y}$

29. (b) $K = \dfrac{F}{x} = \dfrac{4 \times 9.8}{2 \times 10^{-2}} = 19.6 \times 10^2$

Work done $= \dfrac{1}{2} \times 19.6 \times 10^2 \times (0.05)^2 = 2.45\,J$

30. (c) If ℓ is the original length of wire, then change in length of first wire, $\Delta\ell_1 = (\ell_1 - \ell)$

change in length of second wire, $\Delta\ell_2 = (\ell_2 - \ell)$

Now, $Y = \dfrac{T_1}{A} \times \dfrac{\ell}{\Delta\ell_1} = \dfrac{T_2}{A} \times \dfrac{\ell}{\Delta\ell_2}$

or $\dfrac{T_1}{\Delta\ell_1} = \dfrac{T_2}{\Delta\ell_2}$ or $\dfrac{T_1}{\ell_1 - \ell} = \dfrac{T_2}{\ell_2 - \ell}$

or $T_1\ell_2 - T_1\ell = T_2\ell_1 - \ell T_2$ or $\ell = \dfrac{T_2\ell_1 - T_1\ell_2}{T_2 - T_1}$

31. (a) $\delta = \dfrac{W\ell^3}{3\,YI}$, where W = load, ℓ = length of beam and I is geometrical moment of inertia for rectangular beam,

$I = \dfrac{bd^3}{12}$ where b = breadth and d = depth

For square beam b = d

$\therefore\ I_1 = \dfrac{b^4}{12}$

For a beam of circular cross-section, $I_2 = \left(\dfrac{\pi r^4}{4} \right)$

$\therefore\quad \delta_1 = \dfrac{W\ell^3 \times 12}{3\,Y\,b^4} = \dfrac{4\,W\ell^3}{Y\,b^4}$ (for sq. cross section)

and $\delta_2 = \dfrac{W\ell^3}{3\,Y(\pi r^4/4)} = \dfrac{4\,W\ell^3}{3Y(\pi r^4)}$

(for circular cross-section)

Now $\dfrac{\delta_1}{\delta_2} = \dfrac{3\pi r^4}{b^4} = \dfrac{3\pi r^4}{(\pi r^2)^2} = \dfrac{3}{\pi}$

($\because b^2 = \pi r^2$ i.e., they have same cross-sectional area)

32. (a) Compressibility $= \dfrac{1}{\text{Bulk modulus}}$

As bulk modulus is least for ethanol (0.9) and maximum for mercury (25) among ehtanol, mercury and water.

Hence compression in volume $\dfrac{\Delta V}{V}$

Ethanol > Water > Mercury

33. (c) The given graph does not obey Hooke's law. and there is no well defined plastic region. So the graph represents elastomers.

34. (a) $U / \text{volume} = \dfrac{1}{2} Y \times \text{strain}^2 = 3600\ J\,m^{-3}$

[Strain $= 0.06 \times 10^{-2}$]

35. (d) Potential energy per unit volume of the wire is given by :

$u = \dfrac{1}{2} \dfrac{(\text{Stress})^2}{\text{Young's modulus}} = \dfrac{1}{2} \dfrac{S^2}{Y}$

As stress, $S = \dfrac{\text{Force}}{\text{Area}}$

$\therefore\ \dfrac{S_1}{S_2} = \left(\dfrac{F_1}{F_2} \right)\left(\dfrac{A_2}{A_1} \right)$

As $F_1 = F_2$ (Given)

$\therefore\ \dfrac{S_1}{S_2} = \left(\dfrac{F_1}{F_2} \right)\left(\dfrac{A_2}{A_1} \right) = \left(\dfrac{A_2}{A_1} \right)$ ..(i)

The two wires are of the same material, therefore their Young's moduli will be same i.e., $Y_1 = Y_2$

$\therefore\ \dfrac{u_1}{u_2} = \left(\dfrac{S_1}{S_2} \right)^2 = \left(\dfrac{A_2}{A_1} \right)^2$

$= \left(\dfrac{\pi\left(\dfrac{d_2}{2} \right)^2}{\pi\left(\dfrac{d_1}{2} \right)^2} \right) = \left[\left(\dfrac{d_2}{d_1} \right)^2 \right]^2$

$= \left(\dfrac{d_2}{d_1} \right)^4 = \left(\dfrac{2}{1} \right)^4 = \dfrac{16}{1}$ $\left(\because \dfrac{d_1}{d_2} = \dfrac{1}{2}\,(\text{Given}) \right)$

36. (b) Using Hooke's law, $F = kx$ we can write

$\qquad 4 = k(a - \ell_0)$ …(i)

and $\qquad 5 = k(b - \ell_0)$ …(ii)

If ℓ be the length under tension 9N, then

$\qquad 9 = k(\ell - \ell_0)$ …(iii)

After solving above equations, we get

$\qquad \ell = (5b - 4a)$.

37. (b) $F = Y \times A \times \dfrac{l}{L} \Rightarrow F \propto r^2$ (Y, l and and L are constant)

If diameter is made four times then force required will be 16 times, i.e., $16 \times 10^3\ N$

38. (c) Young's modulus of elasticity is

$Y = \dfrac{F/A}{\Delta L/L}$

$\therefore \Delta L = \dfrac{FL}{AY}$

So, $\Delta L \propto \dfrac{L}{A}$

$\therefore \dfrac{\Delta L_2}{\Delta L_1} = \dfrac{L_2}{L_1} \times \dfrac{A_1}{A_2} = \dfrac{2}{1} \times \dfrac{2}{1} = 4$

$\Delta L_2 = 4 \times \Delta L_1 = 4 \times 1 = 4\ cm$

39. (a) $y = \dfrac{F/A}{\Delta l/l} = \dfrac{F}{A} \cdot \dfrac{l}{\Delta l}$

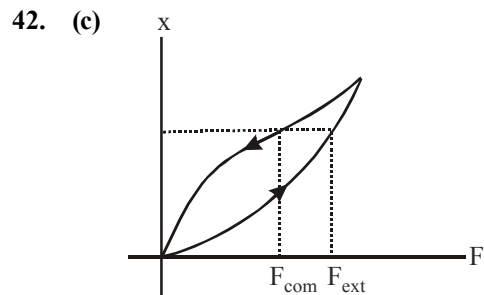$= \dfrac{20 \times 1}{10^{-6} \times 10^{-4}} = 2 \times 10^{11} \, \text{Nm}^{-2}$

40. (d) $K = \dfrac{\Delta P}{\Delta V/V} = \dfrac{h\rho g}{\Delta V/V} = \dfrac{200 \times 10^3 \times 10}{0.1/100} = 2 \times 10^9$

41. (d) Bulk Modulus $= \dfrac{dp}{\dfrac{dv}{v}}$

$dp = h\rho g = 200 \times 10^3 \times 9.8$

$\dfrac{dv}{v} = \dfrac{0.1}{100}$

Bulk modulus $= \dfrac{200 \times 10^3 \times 9.8}{0.1/100} = 19.6 \times 10^8 \, \text{N/m}^2$

42. (c)

From the figure, it is clear that
$F_{com} < F_{ext.}$

43. (d) $\dfrac{\Delta r/r}{\Delta l/l} = 0.5 = \dfrac{1}{2}, \dfrac{\Delta r}{r} = \dfrac{1}{2}\dfrac{\Delta l}{l}$

44. (b) As $Y = \dfrac{\dfrac{F}{A}}{\dfrac{\Delta l}{l}} \Rightarrow \Delta l = \dfrac{Fl}{AY}$

But $V = Al$ so $A = \dfrac{V}{l}$

Therefore $\Delta l = \dfrac{Fl^2}{VY} \propto l^2$

Hence graph of Δl versus l^2 will give a straight line.

45. (b) $K = \dfrac{F}{x} = \dfrac{4 \times 9.8}{2 \times 10^{-2}} = 19.6 \times 10^2$

Work done $= \dfrac{1}{2} \times 19.6 \times 10^2 \times (0.05)^2 = 2.45 \, J$